

## Effect of Slope Prediction Methods on Slope and Erosion Estimates

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### Abstract

GIS can be used to reduce data collection demands by extracting valuable information from existing data bases. One important application is in estimating slope steepness, which is a critical factor in estimating soil loss and chemical movement. Four algorithms/techniques that have been widely used to predict slope from elevation data sets, such as those found in raster-based GIS, are described. Using two 10X10 cell data sets, the slope and USLE LS-factor were estimated for each grid cell using each method and then compared. Estimates of slopes for these areas were also obtained from topographic maps and site observation. Significant differences in the estimated slopes between methods were found. The effects of slope estimation techniques on non-point source models are demonstrated and discussed.

### Introduction

Computer models are widely used for estimating the impacts of management and land use change on soil and water resources. However, one of the greatest limitations of some hydrologic and water quality models is their need for large input data sets, requiring significant time, effort and money to collect. One way of reducing this problem is by extracting the input data required by these models from data bases. The emerging technology called Geographic Information Systems (GIS), defined as a tool to collect, manage, store and display spatially varying

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data, can be used effectively to provide this data, thereby creating the data sets required by hydrologic and water quality models. In addition, the data in the GIS have many other applications.

Slope steepness is one of the most important and most widely used topographic attributes. Many land capability classification systems use slope as the primary means of describing class, along with other factors such as soil depth, soil fertility and soil permeability. In addition, several distributed parameter hydrologic and water quality models that have been developed are based on the grid-cell concept requiring slope steepness information for each cell. Using this concept, a catchment is partitioned into a series of square cells with soil, morphologic (i.e. slope, slope length, aspect), vegetation and land use properties assigned to each cell. A raster based GIS divides areas into square grids for data storage, analysis, and manipulation. The models that use this grid-format include the Areal Nonpoint Source Watershed Environment Response Simulation model-ANSWERS (Beasley and Huggins, 1982), the AGricultural Non-Point Source pollution model-AGNPS (Young et al., 1985), the Systeme Hydrologique European model-SHE (Abbott et al., 1986) and the grid Water Erosion Prediction Project model-WEPP (Foster et al., 1987).

The focus of this paper is to make readers aware of differences obtained from slope estimation methods commonly used with GIS elevation layers. Four algorithms/techniques that have been widely used to predict slope from elevation data sets are discussed and compared. To show the variability of these techniques, two 10X10 matrix data sets were used to estimate the slope for each grid using the four methods, the results compared, and the significance of these differences demonstrated.

## Methods to Predict Slope

### *Neighborhood Method*

This method is used by a raster based GIS tool called GRASS (CERL, 1988) to obtain slope from elevation data sets using a neighborhood operation. A similar method of estimating the slope of a cell is used by OSU MAP (OSU, Department of Geography, 1989), a PC-based GIS tool. The neighborhood method considers the eight neighboring cells and predicts slope for the center cell. Figure 1 shows a schematic 3X3 square grid cell pattern. Let  $z_1 \dots z_9$  be the elevation at the center of each cell and let 'd' be the grid cell side length, and therefore 'd' is the vertical or horizontal distance between the centers of two cells.

The east-west slope is given by:

$$S_{e-w} = \frac{(z_3 + 2z_4 + z_5) - (z_1 + 2z_8 + z_7)}{4 * 2 * d} \quad [1]$$

and similarly, north-south slope is given by:

$$S_{n-s} = \frac{(z_1 + 2z_2 + z_3) - (z_7 + 2z_6 + z_5)}{4 * 2 * d} \quad [2]$$

The resultant slope for the center grid is given by:

$$S = \sqrt{S_{e-w}^2 + S_{n-s}^2} \quad [3]$$

One important factor to be noticed is that in this method the elevation at the center ( $z_9$ ) is never considered for estimating the slope, only its neighbors are considered. This leads to inaccurate estimation of average slope values if the elevation data have small pits or ridges. A function to smooth the elevation data sets is normally used before calculating the slopes, thereby eliminating these small pits or ridges.

### *Quadratic Surface Method*

Zevenbergen and Thorne (1987) used a partial quadratic equation to fit a surface that passes exactly through the nine elevation data points ( $z_1 \dots z_9$ ) and is given by:

$$Z = Ax^2y^2 + Bx^2y + Cxy^2 + Dx^2 + Ey^2 + Fxy + Gx + Hy + I \quad [4]$$

The nine parameters A through I can be determined from the nine elevations of the 3X3 submatrix (Figure 1) by Lagrange polynomials (Zevenbergen and Thorne, 1987). The slope is the first derivative of Z with respect to the direction of slope and the slope S is given by:

$$S = \sqrt{G^2 + H^2} \quad [5]$$

where

$$G = \frac{-z_8 + z_4}{2 * d} \quad \text{and} \quad [6]$$

$$H = \frac{z_2 - z_6}{2 * d} \quad [7]$$

This method is widely accepted and used in many Digital Elevation Models (DEMs) as well as in other terrain analysis models (Moore et al., 1988). It considers only the four neighbors and does not consider the elevation of the center cell ( $z_9$ ) for which slope is being calculated. The same caution to eliminate pits or ridges in the elevation data is applicable here.

### *Best Fit Plane Method*

Beasley and Huggins (1982) used a linear equation to fit a plane through the four corner elevation points of a cell to derive the average slope of the cell. Let  $e_1, e_2, e_3$  and  $e_4$  be the four corner elevation data of a square grid cell with a side length of  $d$  units. Then, the weighted average corner elevations are given as:

$$z_1 = \frac{3e_1 + e_2 - e_3 + e_4}{4} \quad [8]$$

$$z_2 = \frac{e_1 + 3e_2 + e_3 - e_4}{4} \quad [9]$$

$$z_3 = \frac{-e_1 + e_2 + 3e_3 + e_4}{4} \quad [10]$$

$$z_4 = \frac{e_1 - e_2 + e_3 + 3e_4}{4} \quad [11]$$

Next, find the maximum of  $z_1, z_2, z_3$  and  $z_4$ , and denote the subscript as **max**. Then:

$$AT = \frac{z_{\max+1} - z_{\max+2}}{z_{\max} - z_{\max+1}} \quad [12]$$

$$Slen = d * \sqrt{1 + AT^2} \quad [13]$$

and slope of the cell is given as:

$$S = \frac{(z_{\max} - z_{\max+2}) AT}{Slen} \quad [14]$$

For example, if **max** is 3 then **max+1** is 4 and **max+2** is 1 and so on. This method differs from the first two methods, since it uses a linear fit through the elevation data and considers only four neighbors for estimating the average slope of a cell.

#### *Maximum Slope Method*

Shanholtz et al. (1990) used a maximum slope technique for calculating the slope of the center cell with respect to its eight neighbors. The estimated slopes were then used to predict soil losses using the Universal Soil Loss Equation (USLE) with the VirGIS (Virginia Geographic Information System) tool. OSU MAP (OSU, Department of Geography, 1989) also has an option to estimate the slope of cells using this method. The slope of the cell at the center

(Figure 1) is calculated by the expression  $S_i = \max |z_9 - z_i| / L_c * 100$ , where  $L_c$  is the distance between neighboring cell midpoints, and  $i$  is cell 1, 2, 3, ... 8. For the adjacent cells ( $i = 2, 4, 6$  and 8),  $L_c$  is the cell width  $d$ , while for the diagonal cells ( $i = 1, 3, 5$  and 7)  $L_c$  is the cell diagonal distance  $\sqrt{2}d$ .

This method is more practical than the others and is applied in the field to estimate the slope at a particular point. Generally, elevation data for watershed models are derived from quad-sheets, where data are represented in terms of fixed contour intervals, and assumed to vary linearly between the contour intervals. For square grids, using this method for estimating slope by comparing the maximum elevation difference per unit length between the center grid and the eight neighbors leads to misrepresentation and tends to over estimate slopes. The size of the square grid is a limiting factor and needs to be carefully selected by looking at the contour maps and their intervals. Small pits or ridges in the data set will also affect slope estimation, resulting in unpredictable results.

#### *Observed Slope Estimation*

In this study, the observed slope values were derived from site observations and USGS topographic quad-sheets. Using quad-sheets a grid was placed on each of the areas of interest and the elevation for each cell was determined by interpolating between contour lines. The slope value for each cell was found by dividing the maximum elevation difference between the cell and its 8 neighbors by the distance between the mid-points of those two cells. Finally, the slope was averaged over the test site and considered as the test site "observed value". A site observation conducted for the test sites found that the quad-sheet observed slope values of the sites were in close agreement with the site observations.

## **An Example Application**

The four methods of estimating slopes were applied to elevation data sets derived from aerial photography with a scale of 1:40,000 for the Indian Pine Natural Resources Station located in Tippecanoe County, Indiana near Purdue University. Two areas were selected for evaluating/demonstrating the different slope estimation methods, each with a 10X10 matrix of square grids with side lengths of 30 meters (98.42 ft). One area was relatively flat (average slope approximately 1%) and the other relatively steep (average slope approximately 16%). The four methods were applied to each area and the slopes for each grid cell estimated.

From the 10X10 sample data set, the slopes were estimated for the 8X8 grid cell leaving the first and last row, and first and last column grids. These rows and columns were not included so that edge effects would not bias the comparisons. The results are shown in Tables 1 and 2 for the flat and steep areas, respectively. The order of the estimated slope method in each grid is neighborhood, quadratic surface, best fit plane and the maximum slope method. The neighborhood method predicted lower values than the other methods and the maximum slope method always predicted the highest value for each grid. The range of slope values between the methods (as a percentage) for each grid was larger in the flat area than that of the steep area.

## **Discussion and Results of Slope Estimation Methods**

Slope plays a major role in estimating runoff, erosion, chemical movement and soil losses. To study the effect of slope estimation method on soil loss predictions, the LS-factor of the Universal Soil Loss Equation-USLE (Wischmeier and Smith, 1978) was estimated for each grid cell using the the following equation (Wischmeier and Smith, 1978):

$$LS = (\lambda / 72.6)^m ( 65.41 \sin^2 \theta + 4.56 \sin \theta + 0.065 ) \quad [15]$$

where,  $\lambda$  = slope length in feet;

$\theta$  = angle of slope; and

$m$  = 0.5 if the percent of slope is 5 or more, 0.4 on slopes of 3.5 to 4.5 percent,

0.3 on slopes of 1 to 3 percent, and 0.2 on uniform gradients of less than 1 percent.

The average slopes and average LS-factors are shown in Tables 3 and 4 for assumed slope lengths of 100, 150, 200, and 300 feet for the flat and steep areas, respectively. Slope lengths were assumed for demonstration purposes since slope lengths are not easily derived from digital elevation maps. Slope lengths in GIS databases are usually assumed to be a soil property.

In the flat slope areas the value of observed slope was in close agreement with the neighborhood method due to negligible elevation differences. In the steep slope area significant differences existed between the observed and the neighborhood method estimates of slopes. For the observed method, only the contour level difference within cells were accounted for, but in the neighborhood method the neighboring 8 cells were taken into consideration. For the neighborhood method, the ridges and the pits were eliminated before estimating slope for each cell, likely causing some of the differences between estimates of this method and observed method. Of the slope estimation methods tested, the neighborhood method most closely agreed with the observed data.

The variation of slope and LS-factors for assumed lengths among the four slope estimation methods is higher for the relatively flat area than for the steep area. As expected, the four methods are less sensitive to large elevation differences. In both cases, the maximum slope method tends to greatly overestimate slopes. The approximate overestimation of slope by the maximum slope method compared to the neighborhood method was 286% and 80% for flat and steep areas, respectively. Similarly, the LS-factors were also overestimated by 207%, 213%, 231%, 244% for assumed lengths of 100, 150, 200 and 300 feet respectively for the flat area. LS



differences between these two methods were about 161% irrespective of assumed slope lengths for the steep area. The selection of slope estimation techniques for use with the USLE and other applications needs careful attention. As shown in the Tables 3 and 4, using the maximum slope method tends to overestimate USLE prediction of soil erosion by approximately 2 times and 1.6 times in relatively flat and steep areas, respectively. The other two methods for slope estimation tend to overestimate the slope, but to a lesser degree. Moore et al. (1988) found that the relative magnitude of many hydrologic processes operating in natural landscapes are sensitive to topographic position. However, little importance was given to the slope prediction technique when applied to derive slope as an input for various hydrologic and water quality models. To verify or to validate these models, parameters defining the soil and its properties are adjusted to match observed values, even though the variation in predicting the slope can have a wide range as shown in Tables 3 and 4.

The slope values for the individual grids show significant differences between the four methods. Caution is required when using any of the methods, especially the maximum slope method to derive a slope layer for use in a distributed parameter model. These models route runoff, erosion and chemical movement independently for each grid along the drainage path and accumulate results. Hence, the variations in the slope values will affect the results of the models to a greater extent. This was shown by the variation in LS-factor estimation for each of the four slope estimation methods (Tables 3 and 4).

## Summary

Slope plays a vital role in predicting the direction of flow and amounts of non-point source pollution including soil loss and chemical movement. Four of the most common methods (neighborhood, quadratic surface, best fit plane and maximum slope method) used by GIS and other tools for estimating slope from grid elevation data were compared and significant

differences found in the estimated slopes. Two 10X10 cell data sets were selected from fairly flat (average slope of about 1%) and steep areas (average slope of about 16%) to apply the four methods. Observed slope values were estimated using quad-sheets and site observations. In the flat areas the difference was negligible between observed and neighborhood method estimates. In the steep areas there was a more significant difference in average slope values between the observed and neighborhood method. Of the methods tested, the neighborhood method most closely approximates observed slope values. The maximum slope method always estimated higher slopes than the other methods. The neighborhood method tended to predict the lowest slope values. Slope differences between the neighborhood method and the maximum slope method were about 286% and 80% for the flat and the steep areas, respectively. The maximum slope method predicts the USLE LS factor to be about 2 and 1.6 times greater than those for the neighborhood method on the flat and the steep area, respectively. Careful selection of slope prediction method is recommended.

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1	2	3
8	9	4
7	6	5

Figure 1. Schematic of 3X3 square grid cell matrix.

Table 1. Estimated slopes in percent for each grid in the flat area by neighborhood, quadratic surface, best fit plane and the maximum slope methods.

Column/ Row	1	2	3	4	5	6	7	8
1	1.9 <sup>1</sup>	0.9	0.6	1.4	0.2	1.9	1.1	0.8
	2.1 <sup>2</sup>	0.9	1.3	3.0	1.9	2.9	2.4	1.4
	1.5 <sup>3</sup>	1.5	1.6	1.9	1.3	1.9	2.6	1.6
	2.5 <sup>4</sup>	2.8	3.9	6.0	3.5	3.8	4.1	1.8
2	1.8	1.2	1.0	1.3	0.9	2.3	1.3	0.8
	2.0	2.1	3.0	1.4	3.3	2.8	2.4	1.1
	2.4	2.8	0.4	4.4	4.5	2.3	4.1	2.7
	4.8	2.5	3.3	6.0	6.0	6.1	6.1	2.5
3	2.4	0.3	2.0	2.3	0.4	1.7	0.4	0.6
	3.1	0.8	3.2	3.2	1.5	2.5	1.3	0.7
	4.0	3.3	1.9	2.5	4.2	2.1	2.4	2.2
	4.8	4.0	7.0	6.0	4.2	3.5	4.0	1.8
4	2.0	1.6	1.0	0.2	1.4	0.9	0.8	0.4
	2.0	3.2	0.9	1.5	1.8	1.7	1.4	1.1
	4.8	0.8	4.2	3.1	0.5	3.7	1.9	0.6
	4.7	5.8	7.0	4.0	4.5	4.5	4.1	3.2
5	1.6	1.2	0.8	0.8	1.1	0.2	0.5	0.4
	2.1	1.7	2.3	1.2	1.4	1.2	0.7	1.8
	4.6	0.5	4.4	3.3	0.7	3.1	3.8	1.4
	4.4	4.4	6.3	2.8	2.5	3.2	4.5	4.5
6	0.4	1.6	1.2	0.4	1.5	1.5	1.3	0.3
	0.1	2.2	1.6	0.7	1.0	2.2	1.6	1.0
	4.4	3.0	3.2	1.6	1.0	1.4	2.7	1.0
	4.4	4.2	2.3	1.6	3.2	3.9	3.4	3.2
7	0.6	1.7	0.8	0.2	1.7	0.8	0.7	0.5
	0.9	2.2	1.0	0.1	2.9	1.3	1.8	0.6
	3.5	4.1	1.3	0.4	1.0	3.6	3.1	1.6
	3.9	3.9	1.8	1.0	6.2	6.2	4.6	2.6
8	0.7	1.3	0.7	0.2	1.0	1.2	0.6	1.0
	1.8	1.8	0.9	0.1	0.1	3.0	1.9	0.1
	3.8	3.5	2.4	1.0	0.8	3.4	4.2	0.7
	5.3	3.0	1.9	1.3	3.3	5.8	4.1	4.4

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- 1 Neighborhood Method
  - 2 Quadratic Surface Method
  - 3 Best Fit Plane Method
  - 4 Maximum Slope Method

Table 2. Estimated slopes in percent for each grid in the steep area by neighborhood, quadratic surface, best fit plane and the maximum slope methods.

Column/ Row	1	2	3	4	5	6	7	8
1	28.8 <sup>1</sup>	14.6	18.0	20.2	14.8	33.3	13.5	19.7
	26.2 <sup>2</sup>	16.5	20.2	27.0	10.0	40.9	13.6	18.0
	34.8 <sup>3</sup>	26.0	23.9	39.9	40.9	37.7	43.0	23.7
	32.7 <sup>4</sup>	27.9	34.4	45.4	38.9	51.0	51.0	42.9
2	27.8	13.8	5.6	2.0	8.4	24.3	20.2	22.9
	28.9	10.8	6.3	5.0	10.7	21.8	28.1	22.4
	32.1	23.2	6.0	7.6	4.3	18.3	42.9	36.1
	42.7	21.3	6.2	12.5	15.1	37.2	39.2	34.9
3	34.0	19.9	6.3	2.2	7.6	17.4	20.0	17.3
	33.6	18.4	5.2	2.2	7.7	15.1	26.3	15.6
	35.6	23.8	5.8	4.9	8.7	15.2	32.8	29.3
	39.7	35.7	9.9	8.7	14.6	32.7	35.4	39.2
4	33.6	25.1	13.4	5.8	2.2	6.8	8.2	7.5
	39.7	31.7	14.7	2.0	2.0	7.1	11.2	2.9
	0.0	39.0	11.4	3.6	1.3	6.6	15.6	12.7
	41.8	41.8	25.5	19.4	5.9	15.1	16.5	17.0
5	30.2	21.0	12.6	9.4	4.5	3.1	2.7	3.3
	23.2	14.1	16.5	14.1	5.2	3.3	4.2	5.0
	32.4	32.0	23.7	19.4	2.5	4.5	6.0	12.3
	29.2	22.9	25.5	25.5	6.7	6.6	5.6	7.9
6	33.2	20.9	12.2	14.1	9.1	5.4	8.2	10.9
	36.4	26.3	12.0	11.5	8.8	1.4	10.5	11.5
	27.1	32.5	19.1	18.2	6.3	6.0	3.2	5.8
	42.9	42.9	36.7	30.3	18.8	15.2	18.1	25.0
7	35.7	31.7	25.0	21.4	9.4	9.7	11.5	12.3
	41.3	30.3	28.1	16.7	13.8	18.9	14.4	14.8
	33.4	43.7	26.9	31.2	19.4	15.3	11.7	21.3
	39.7	48.3	36.7	30.6	32.7	20.8	20.8	25.0
8	22.9	22.1	26.4	32.4	11.9	8.5	12.4	3.9
	17.6	26.1	32.1	42.3	11.8	5.6	14.4	7.4
	19.5	48.3	26.5	40.7	28.2	17.0	17.1	11.0
	48.3	35.8	55.1	55.1	32.7	14.4	15.8	15.8

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- 1 Neighborhood Method  
 2 Quadratic Surface Method  
 3 Best Fit Plane Method  
 4 Maximum Slope Method

Table 3. Average slopes and LS-factors of observed and four slope estimation methods for the flat area.

<i>Method</i>	<i>Slope</i>	<i>Assumed Slope Length</i>			
	<i>percent</i>	<i>100'</i>	<i>150'</i>	<i>200'</i>	<i>300'</i>
Observed	0.9	0.12	0.13	0.14	0.15
Neighborhood	1.0	0.13	0.15	0.16	0.18
Quadratic surface	1.7	0.18	0.20	0.22	0.25
Best fit plane	2.5	0.24	0.27	0.30	0.34
Maximum slope	4.0	0.40	0.47	0.53	0.62

Table 4. Average slopes and LS-factors of observed and four slope estimation methods for the steep area.

<i>Method</i>	<i>Slope</i>	<i>Assumed Slope Length</i>			
	<i>percent</i>	<i>100'</i>	<i>150'</i>	<i>200'</i>	<i>300'</i>
Observed	11.6	1.72	2.10	2.43	2.97
Neighborhood	15.8	2.79	3.42	3.95	4.83
Quadratic surface	16.9	3.10	3.80	4.38	5.37
Best fit plane	21.7	4.67	5.71	6.60	8.08
Maximum slope	28.4	7.28	8.91	10.29	12.61